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On graviphoton F-terms of $\mathcal{N}=1$ SU(N) SYM with fundamental matter

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Abstract: We consider $\mathcal{N}=1$ SU(N_c) supersymmetric gauge theory with chiral matter multiplets in the fundamental representation of the gauge group. The general form of the meson correlation functions in the presence of graviphoton background with or without gravity is obtained. Finally, the perturbation theory scheme of computing these correlation functions is discussed.

Keywords: Supersymmetric Effective Theories.

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Contents

1. Introduction

F-terms of four-dimensional supersymmetric gauge theories in supergravity and graviphoton backgrounds have attracted much attention in recent years. On the one hand they are related to certain exactly computable amplitudes of two gravitons and graviphotons. On the other hand they are computed by second quantized partition functions of topological strings [1], and have an interesting mathematical structure [2]. Gravitational F-terms are directly related to the partition function of two-dimensional non-critical strings [3, 4]. Recently, gravitational F-terms have been related to the computation of certain $\mathcal{N} = 2$ black hole partition functions [5].

In this paper we will consider the gravitational and graviphoton correlation functions in the context of four-dimensional $\mathcal{N} = 1$ supersymmetric gauge theories. Dijkgraaf and Vafa suggested a matrix model description, where the gravitational F-terms can be computed by summing up the non-planar matrix diagrams [6]. The assumption made is that the

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relevant fields are the glueball superfields S_i and the F-terms are holomorphic couplings of the glueball superfields to gravity and the graviphoton. The DV matrix proposal has been proven diagrammatically in [7, 8].

The gravitational and graviphoton F-terms of interest are of the form

$$\Gamma_1 = \sum_{g=0}^{\infty} \int d^4x d^2\theta \left(F_{\alpha\beta}F^{\alpha\beta}\right)^g N_i \frac{\partial F_g(S)}{\partial S_i}, \tag{1.1}$$

$$\Gamma_2 = \sum_{g=1}^{\infty} g \int d^4x d^2\theta G^2 \left(F_{\alpha\beta}F^{\alpha\beta}\right)^{g-1} F_g(S), \tag{1.2}$$

where $G_{\alpha\beta\gamma}$ is the $\mathcal{N}=1$ Weyl superfield and $F_{\alpha\beta}$ is the graviphoton. According to the DV proposal, $F_g(S_i)$ is the partition function of the corresponding matrix model evaluated by summing the genus g diagrams with S_i being the 't Hooft parameters.

The approach we will take is to use the generalized Konishi anomaly equations and some knowledge on the correlation functions to obtain information about the perturbative contribution to the correlation functions involving gravity and the graviphoton. In general, it is not clear in which cases the generalized Konishi anomaly equations are sufficient in order to determine the gravitational and graviphoton F-terms. We will study $\mathcal{N}=1$ SU(N_c) supersymmetric gauge theory with fundamental matter and show that the anomaly equations are insufficient to obtain the full perturbative correlation functions (and the F-terms). We will also discuss the field theoretic perturbative diagrammatic computation.

Other recent works on the computation of gravitational and graviphoton correlation functions and F-terms are [9–16].

The paper is organized as follows. In section 2 we review the computational scheme. Then in section 3 we apply the scheme to the computation of correlation functions in SU(N_c) SYM. Later we demonstrate the problems with the diagrammatic computation in section 4.

2. The computational scheme

Here we review the scheme used for computing the correlation functions in the presence of either graviphoton or gravity backgrounds.

2.1 The chiral ring

We consider here an $\mathcal{N}=1$ supersymmetric gauge theory with chiral matter multiplets coupled to it. Chiral operators are operators annihilated by the covariant derivative $\bar{D}_{\dot{\alpha}}$. All such operators modulo terms which are $\bar{D}_{\dot{\alpha}}$ exact form the ring structure of the chiral ring.

Denoting by $W_\alpha = -\frac{1}{4}\bar{D}^2 e^{-V} D_\alpha e^V$ the spinor field-strength superfield of the vector superfield V one has in flat space that in the chiral ring (i.e., up to $\bar{D}_{\dot{\alpha}}$ exact terms)

$$\{W_\alpha, W_\beta\} = 0\,. \tag{2.1}$$

This relation is modified in a background of gravity and the graviphoton field. Let $G_{\alpha\beta\gamma}$ be the $\mathcal{N}=1$ Weyl superfield and $F_{\alpha\beta}$ be the graviphoton field which together form the $\mathcal{N}=2$ Weyl superfield $H_{\alpha\beta}=F_{\alpha\beta}+\hat{\theta}^{\gamma}G_{\alpha\beta\gamma}$, where $\hat{\theta}$ is the additional supercoordinate of $\mathcal{N}=2$ superspace. In the presence of these either the supercoordinates become non-anti-commutative or (2.1) is modified [7, 8, 17] to

$$\{W_\alpha, W_\beta\} = F_{\alpha\beta} + 2G_{\alpha\beta\gamma}W^\gamma\,. \tag{2.2}$$

This deformation of the chiral ring leads to the chiral ring relations [12]

$$\begin{aligned}
[W_\alpha, W^2] &= -2F_{\alpha\beta}W^\beta\,, && (2.3)\\
\{W_\alpha, W^2\} &= -\frac{2}{3}\left(G^2 W_\alpha + G_{\alpha\beta\gamma}F^{\beta\gamma}\right)\,, && (2.4)\\
W_\alpha W^2 &= -F_{\alpha\beta}W^\beta - \frac{1}{3}G^2W_\alpha - \frac{1}{3}G_{\alpha\beta\gamma}F^{\beta\gamma}\,, && (2.5)\\
(G^2)^2 &= 0\,. && (2.6)
\end{aligned}$$

In addition, for a chiral superfield in the fundamental or anti-fundamental representation [18]

$$W_{\alpha a}{}^{b}Q^i_b = \tilde{Q}^a_i W_{\alpha a}{}^{b} = 0 \tag{2.7}$$

in the chiral ring.

2.2 The Konishi anomaly equations

The classical Konishi equations in the chiral ring for a field transformation δQ^i_a are

$$\frac{\partial W_{\rm tree}}{\partial Q^i_a}\delta Q^i_a = 0\,. \tag{2.8}$$

These are modified quantum mechanically [19, 20] to

$$\left\langle \frac{\partial W_{\rm tree}}{\partial Q^i_a}\delta Q^i_a \right\rangle + \left\langle \left(\frac{1}{32\pi^2}W_{\alpha a}{}^{b}W^{\alpha}{}_{b}{}^{c} + \frac{1}{32\pi^2}\frac{G^2}{3}\delta^c_a\right)\frac{\partial \delta Q^i_c}{\partial Q^i_a}\right\rangle = 0\,. \tag{2.9}$$

As argued in [12], the Konishi anomaly equations are not modified in the presence of the graviphoton field. The argument is based on the graviphoton being of dimension three so any Lorentz scalar with smooth limits of the dimensional parameters of the theory constructed from it would have a dimension greater then three. All the terms in the Konishi anomaly equation are of dimension three, therefore the anomaly equation for a field transformation δQ^i_a remains of the form (2.9) and is not modified.

In the next section we will use the Konishi anomaly equations in order to obtain perturbative information on the correlation functions. As will be seen, the Konishi equations are not sufficient to completely determine the correlations functions, but they do provide some constraints on their general form.

3. $\mathrm{SU}(N_c)$ SYM with fundamental matter

In this section we consider the $\mathcal{N}=1$ $\mathrm{SU}(N_c)$ SYM theory with chiral matter multiplets Q_a^i and $\tilde{Q}_i^a$ ($a,b,\ldots=1,\ldots,N_c$ are color indices and $i,j,\ldots=1,\ldots,N_f$ are flavor indices) in the fundamental and anti-fundamental representation, respectively, considered in [21, 22]. The theory has the tree-level superpotential

$$W_{\mathrm{tree}} = m\,\mathrm{tr}\,M + \lambda\,\mathrm{tr}\,M^2\,, \tag{3.1}$$

where $M_i{}^j = \tilde{Q}_i^a Q_a^j$ are the gauge-invariant meson operators. For simplicity we will take the case of a single flavor ($N_f = 1$), but the results should be readily extendable to $N_f > 1$.

3.1 The anomaly equations

We first look at the field transformation $\delta Q_a = Q_a M^k$. It yields the equations obtained in [22]

$$m\left\langle M^{k+1}\right\rangle + 2\lambda\left\langle M^{k+2}\right\rangle + \frac{N_c+k}{3}G^2\left\langle M^k\right\rangle = S\left\langle M^k\right\rangle. \tag{3.2}$$

(Here and henceforth we redefine the Weyl superfield $G^2 \to G^2/32\pi^2$.)

Since the graviphoton does not appear explicitly in the anomaly equation, the only way to obtain graviphoton dependence is via the chiral ring relations (2.3)–(2.5) or by including it in the transformation δQ_a. However, using Lorentz invariant graviphoton terms such as $F_{\alpha\beta}F^{\alpha\beta}$ in the transformation will only result in multiplying the entire anomaly equation by these expressions and not yield any new independent equations.

δQ should be a scalar in the fundamental representation of the gauge group, so W_α can be incorporated into it only as $W_{\alpha a}{}^b W^\alpha{}_b{}^c Q_c$, which vanishes in the chiral ring, or as $S = -\frac{1}{32\pi^2}\,\mathrm{Tr}(W_\alpha W^\alpha)$ which will only multiply the equation by S. Another possible scalar can be constructed using the graviphoton: $F_{\alpha\beta}W^\alpha W^\beta$. By using the symmetry of $F_{\alpha\beta}$ and (2.2) one has

$$F_{\alpha\beta}W^\alpha W^\beta = \frac{1}{2}F_{\alpha\beta}F^{\alpha\beta} + F_{\alpha\beta}G^{\alpha\beta}{}_\gamma W^\gamma\,. \tag{3.3}$$

This is in the adjoint representation so an appropriate term can be obtained by acting with it on Q which vanishes in the chiral ring due to (2.7) or by tracing over the gauge indices, making the second term vanish as W_α is traceless.

In general, transformations yielding the graviphoton will either vanish because of (2.7) or will lead to dependent anomaly equations. This is unlike the case of the theory with matter in the adjoint representation of the gauge group since in that case transformations such as $W_\alpha\Phi$ do not vanish and can combine with the W^2 term in the anomaly equation to generate coupling to the graviphoton by the chiral ring relations.

3.2 Correlation functions without gravity

3.2.1 Solution of the anomaly equations

The generalized Konishi anomaly equations for the theory considered were found in the presence of gravity but no graviphoton backgrounds in [22] and with gravity turned off are of the form

$$S\left\langle M^k\right\rangle = m\left\langle M^{k+1}\right\rangle + 2\lambda\left\langle M^{k+2}\right\rangle. \tag{3.4}$$

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Since the graviphoton background does not modify the anomaly equations [12], these remain valid even with the graviphoton turned on.

By performing a z-transform of (3.4) one obtains the single equation

$$S \sum_{k=0}^{\infty} \frac{1}{k!} \left\langle M^k \right\rangle z^k = m \sum_{k=0}^{\infty} \frac{1}{k!} \left\langle M^{k+1} \right\rangle z^k + 2\lambda \sum_{k=0}^{\infty} \frac{1}{k!} \left\langle M^{k+2} \right\rangle z^k . \tag{3.5}$$

We now define the generating function for the meson operator correlation functions

$$f(z) = \sum_{k=0}^{\infty} \frac{1}{k!} \left\langle M^k \right\rangle z^k . \tag{3.6}$$

It follows immediately that

$$\frac{df(z)}{dz} = \sum_{k=0}^{\infty} \frac{1}{k!} \left\langle M^{k+1} \right\rangle z^k , \tag{3.7}$$

$$\frac{d^2 f(z)}{dz^2} = \sum_{k=0}^{\infty} \frac{1}{k!} \left\langle M^{k+2} \right\rangle z^k . \tag{3.8}$$

Hence, the infinite set of equations (3.4) can be written as the ordinary differential equation

$$S f(z) = m \frac{df(z)}{dz} + 2\lambda \frac{d^2 f(z)}{dz^2} , \tag{3.9}$$

whose general solution is

$$f(z) = A_+ \exp\left(\frac{-m + \sqrt{m^2 + 8\lambda S}}{4\lambda} z \right) + A_- \exp\left(\frac{-m - \sqrt{m^2 + 8\lambda S}}{4\lambda} z \right) , \tag{3.10}$$

where A_+ and A_- are coefficients which may depend on the couplings as well as the glueball superfield S and the graviphoton. Particularly, note that any possible graviphoton dependence may enter through these coefficients alone.

Thus, we conclude that the correlation functions are of the form

$$\left\langle M^k \right\rangle = A_+ \left(\frac{-m + \sqrt{m^2 + 8\lambda S}}{4\lambda} \right)^k + A_- \left(\frac{-m - \sqrt{m^2 + 8\lambda S}}{4\lambda} \right)^k . \tag{3.11}$$

Because correlation functions of chiral operators factorize in the absence of the graviphoton and gravity, in the limit $F_{\alpha\beta} \to 0$ either $A_+ \to 1$ and $A_- \to 0$ or the other way around depending on whether the Higgsed vacuum is considered or not. Hence, in the un-Higgsed vacuum

$$A_+ = 1 + O(F_{\alpha\beta} F^{\alpha\beta}) , \qquad A_- = O(F_{\alpha\beta} F^{\alpha\beta}) \tag{3.12}$$

and

$$A_+ = O(F_{\alpha\beta} F^{\alpha\beta}) , \qquad A_- = 1 + O(F_{\alpha\beta} F^{\alpha\beta}) \tag{3.13}$$

in the Higgsed vacuum.

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	U(1)$_Q$	U(1)$_{\tilde{Q}}$
Q	1	0
$\tilde{Q}$	0	1
S	0	0
m	-1	-1
λ	-2	-2
$F_{\alpha\beta}$	0	0

Table 1: $U(1)_Q$ and $U(1)_{\tilde{Q}}$ charges of the various fields and couplings.

3.2.2 The form of $A_\pm$

As noted before, the graviphoton dependence can only enter via the coefficients $A_\pm$. In general, these should depend on the couplings m and λ and on the background fields S and $F_{\alpha\beta}$. Hence, the $A_\pm$ should be sums of terms of the form

$$\lambda^n m^p S^q \left(F_{\alpha\beta}F^{\alpha\beta}\right)^r .$$

From holomorphicity we expect n, p, q and r to be integers. Also since the coefficients are dimensionless, the powers must satisfy

$$-n + p + 3q + 6r = 0\,. \tag{3.14}$$

The limit $F_{\alpha\beta} \to 0$ must be regular so that the ordinary correlation functions without graviphoton background obtained in [21] are recovered. Thus $r \geq 0$. The classical limit $S \to 0$, in which the Konishi anomaly vanishes, must be smooth so $q \geq 0$. In the limit of $\lambda \to 0$ the correlation function $\langle M \rangle$ is either smooth for the case of the un-Higgsed vacuum (the one corresponding to the plus sign solution) or diverges as $1/\lambda$ in the Higgsed vacuum (the minus sign vacuum). Thus, we do not expect additional, higher order divergence as $\lambda \to 0$ so $n \geq 0$.

Finally, the flavor symmetries U(1)$_Q$ and U(1)$_{\tilde{Q}}$ are broken at tree-level by the superpotential. These can be restored by assigning charges to the couplings as given in table 1.

Requiring the correlation functions to be invariant under these restored symmetries yields

$$2n + p = 0\,. \tag{3.15}$$

Putting all of this together we have

$$n = q + 2r \tag{3.16}$$

and the terms in the power series expansion of $A_\pm$ are of the form

$$\left(\frac{\lambda}{m^2}\right)^{q+2r} (F_{\alpha\beta}F^{\alpha\beta})^r S^q\,. \tag{3.17}$$

It should be noted that only terms which vanish in the limit $m \to \infty$ are allowed — in accordance with one's expectation that the matter completely decouples in this limit, leaving only the pure gauge theory coupled to gravity and to the graviphoton.

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3.3 Correlation functions with gravity and graviphoton backgrounds

3.3.1 Solution of the anomaly equations

By performing a z-transform on (3.2) the following equation is obtained

$$S\sum_{k=0}^{\infty}\frac{z^k}{k!}\left\langle M^k\right\rangle = m\sum_{k=0}^{\infty}\frac{z^k}{k!}\left\langle M^{k+1}\right\rangle + 2\lambda\sum_{k=0}^{\infty}\frac{z^k}{k!}\left\langle M^{k+2}\right\rangle + \frac{N_c}{3}G^2\sum_{k=0}^{\infty}\frac{z^k}{k!}\left\langle M^k\right\rangle + \\ +\frac{1}{3}G^2\sum_{k=0}^{\infty}\frac{kz^k}{k!}\left\langle M^k\right\rangle . \tag{3.18}$$

Using the generating function (3.6) one finds that

$$z\frac{df(z)}{dz} = \sum_{k=0}^{\infty}\frac{kz^k}{k!}\left\langle M^k\right\rangle \tag{3.19}$$

and together with the relations (3.7) and (3.8) (3.2) can be cast into the ordinary differential equation

$$Sf(z) = m\frac{df(z)}{dz} + 2\lambda\frac{d^2f(z)}{dz^2} + \frac{N_c}{3}G^2f(z) + \frac{1}{3}G^2z\frac{df(z)}{dz} . \tag{3.20}$$

The function $f(z)$ can be expanded in powers of G^2 using the chiral ring relation $(G^2)^2 = 0$ [12, 11]

$$f(z) = f_0(z) + G^2f_1(z) \tag{3.21}$$

and upon substitution in (3.20) we get two differential equations for $f_0(z)$ and $f_1(z)$:

$$Sf_0(z) = m\frac{df_0(z)}{dz} + 2\lambda\frac{d^2f_0(z)}{dz^2} , \tag{3.22}$$

$$Sf_1(z) = m\frac{df_1(z)}{dz} + 2\lambda\frac{d^2f_1(z)}{dz^2} + \frac{N_c}{3}f_0(z) + \frac{1}{3}z\frac{df_0(z)}{dz} . \tag{3.23}$$

Defining

$$\alpha_{\pm} = \frac{-m \pm \sqrt{m^2 + 8\lambda S}}{4\lambda} , \tag{3.24}$$

the solution of the equation for $f_0(z)$ is (3.10)

$$f_0(z) = A_+e^{\alpha_+z} + A_-e^{\alpha_-z} . \tag{3.25}$$

Plugging this in (3.23) the following equation is obtained

$$Sf_1 = m\frac{df_1}{dz} + 2\lambda\frac{d^2f_1}{dz^2} + \frac{N_c}{3}\left(A_+e^{\alpha_+z} + A_-e^{\alpha_-z}\right) + \\ +\frac{1}{3}z\left(\alpha_+A_+e^{\alpha_+z} + \alpha_-A_-e^{\alpha_-z}\right) . \tag{3.26}$$

By considering a solution of the form

$$f_1(z) = c_+(z)e^{\alpha_+z} + c_-(z)e^{\alpha_-z} \tag{3.27}$$

and further assuming that the equation thus obtained can be divided into separate equations for the unknown functions $c_\pm(z)$ we have

$$2\lambda\frac{d^2c_\pm}{dz^2}+(m+4\lambda\alpha_\pm)\frac{dc_\pm}{dz}+(m\alpha_\pm+2\lambda\alpha_\pm^2-S)c_\pm+\frac{N_c}{3}A_\pm+\frac{1}{3}z\alpha_\pm A_\pm=0\,, \tag{3.28}$$

whose solution is given by

$$\begin{aligned}c_\pm(z)=&\pm\frac{C_{1\pm}}{\sqrt{m^2+8\lambda S}}-\frac{N_c}{3}A_\pm\left(\pm\frac{z}{\sqrt{m^2+8\lambda S}}-\frac{2\lambda}{m^2+8\lambda S}\right)-\\&-\frac{1}{6}\alpha_\pm A_\pm\left[\pm\frac{8\lambda^2}{(m^2+8\lambda S)^{3/2}}-\frac{4\lambda z}{m^2+8\lambda S}\pm\frac{z^2}{\sqrt{m^2+8\lambda S}}\right]+\\&+C_{2\pm}\exp\left(\mp\frac{\sqrt{m^2+8\lambda S}}{2\lambda}z\right),\end{aligned} \tag{3.29}$$

where $C_{1\pm}$ and $C_{2\pm}$ are integration constants which may depend on the couplings, the glueball superfield and the graviphoton.

3.3.2 Constraints on the coefficients

As shown in [8], there are two types of related effective F-terms coupling the glueball to gravity and the graviphoton

$$\Gamma_1=\int d^4xd^2\theta W_0=\sum_{g=0}^{\infty}\int d^4xd^2\theta(F_{\alpha\beta}F^{\alpha\beta})^gN_i\frac{\partial F_g(S)}{\partial S_i}\,, \tag{3.30}$$

$$\Gamma_2=\int d^4xd^2\theta G^2W_1=\sum_{g=1}^{\infty}g\int d^4xd^2\theta G^2(F_{\alpha\beta}F^{\alpha\beta})^{g-1}F_g(S)\,. \tag{3.31}$$

Since the function $F_g(S)$ is found in both, in the case of unbroken gauge symmetry the two are related as

$$\frac{\partial W_0}{\partial u}=N_c\frac{\partial W_1}{\partial S}\,, \tag{3.32}$$

where we have set $u=F_{\alpha\beta}F^{\alpha\beta}$.

The correlation functions $\langle M\rangle$ and $\langle M^2\rangle$ can be obtained from the effective superpotential $W_{\text{eff}}=W_0+W_1G^2$ by differentiating it with respect to the couplings,

$$\langle M\rangle=\frac{\partial W_{\text{eff}}}{\partial m}\,,\quad \langle M^2\rangle=\frac{\partial W_{\text{eff}}}{\partial\lambda}\,. \tag{3.33}$$

Hence, $f_0(z)$ and $f_1(z)$ must satisfy the relations

$$\left.\frac{\partial^2 f_0}{\partial u\partial z}\right|_{z=0}=N_c\left.\frac{\partial^2 f_1}{\partial S\partial z}\right|_{z=0}, \tag{3.34}$$

$$\left.\frac{\partial^3 f_0}{\partial u\partial z^2}\right|_{z=0}=N_c\left.\frac{\partial^3 f_1}{\partial S\partial z^2}\right|_{z=0}. \tag{3.35}$$

The constraint on the integration constants from the first of these is

$$
\begin{aligned}
0 = {} & \frac{2N_c\lambda\left[(1+N_c)m^2+4(2N_c-1)\lambda S-2N_c m\sqrt{m^2+8\lambda S}\right]}{3(m^2+8\lambda S)^{5/2}}A_- - \\
& -\frac{2N_c\lambda\left[(1+N_c)m^2+4(2N_c-1)\lambda S+2N_c m\sqrt{m^2+8\lambda S}\right]}{3(m^2+8\lambda S)^{5/2}}A_+ + \\
& +\frac{N_c m(C_{1-}-C_{1+})}{(m^2+8\lambda S)^{3/2}}+\frac{N_c(C_{2+}-C_{2-})}{\sqrt{m^2+8\lambda S}}+ \\
& +\frac{-m+\sqrt{m^2+8\lambda S}}{4\lambda}\frac{\partial A_+}{\partial u}+\frac{-m-\sqrt{m^2+8\lambda S}}{4\lambda}\frac{\partial A_-}{\partial u}+ \\
& +\frac{N_c\left[-4\lambda S+N_c\left(m^2+8\lambda S+m\sqrt{m^2+8\lambda S}\right)\right]}{6(m^2+8\lambda S)^{3/2}}\frac{\partial A_+}{\partial S}+ \\
& +\frac{N_c\left[4\lambda S-N_c\left(m^2+8\lambda S-m\sqrt{m^2+8\lambda S}\right)\right]}{6(m^2+8\lambda S)^{3/2}}\frac{\partial A_-}{\partial S}+ \\
& +\frac{N_c(m-\sqrt{m^2+8\lambda S})}{4\lambda\sqrt{m^2+8\lambda S}}\frac{\partial C_{1+}}{\partial S}-\frac{N_c(m+\sqrt{m^2+8\lambda S})}{4\lambda\sqrt{m^2+8\lambda S}}\frac{\partial C_{1-}}{\partial S}+ \\
& +\frac{N_c(m+\sqrt{m^2+8\lambda S})}{4\lambda}\frac{\partial C_{2+}}{\partial S}+\frac{N_c(m-\sqrt{m^2+8\lambda S})}{4\lambda}\frac{\partial C_{2-}}{\partial S}\,,
\end{aligned}
\tag{3.36}
$$

while the second one yields

$$
\begin{aligned}
0 = {} & \frac{N_c m\left[(1+2N_c)m(-m+\sqrt{m^2+8\lambda S})+4(1-4N_c)\lambda S\right]A_-}{6(m^2+8\lambda S)^{5/2}}+ \\
& +\frac{N_c m\left[(1+2N_c)m(m+\sqrt{m^2+8\lambda S})-4(1-4N_c)\lambda S\right]A_+}{6(m^2+8\lambda S)^{5/2}}+ \\
& +\frac{2N_c S(C_{1-}-C_{1+})}{(m^2+8\lambda S)^{3/2}}+\frac{N_c(m-\sqrt{m^2+8\lambda S})C_{2-}}{2\lambda\sqrt{m^2+8\lambda S}}-\frac{N_c(m+\sqrt{m^2+8\lambda S})C_{2+}}{2\lambda\sqrt{m^2+8\lambda S}}+ \\
& +\frac{(m+\sqrt{m^2+8\lambda S})^2}{16\lambda^2}\frac{\partial A_-}{\partial u}+\frac{(-m+\sqrt{m^2+8\lambda S})^2}{16\lambda^2}\frac{\partial A_+}{\partial u}+ \\
& +\frac{N_c\left[N_c m^2(m+\sqrt{m^2+8\lambda S})+2(4N_c-1)m\lambda S+2(6N_c+1)\lambda S\sqrt{m^2+8\lambda S}\right]}{12\lambda(m^2+8\lambda S)^{3/2}}\frac{\partial A_-}{\partial S}- \\
& -\frac{N_c\left[N_c m^2(m-\sqrt{m^2+8\lambda S})+2(4N_c-1)m\lambda S-2(6N_c+1)\lambda S\sqrt{m^2+8\lambda S}\right]}{12\lambda(m^2+8\lambda S)^{3/2}}\frac{\partial A_+}{\partial S}+ \\
& +\frac{N_c(m+\sqrt{m^2+8\lambda S})^2}{16\lambda^2\sqrt{m^2+8\lambda S}}\frac{\partial C_{1-}}{\partial S}-\frac{N_c(-m+\sqrt{m^2+8\lambda S})^2}{16\lambda^2\sqrt{m^2+8\lambda S}}\frac{\partial C_{1+}}{\partial S}- \\
& -\frac{N_c(-m+\sqrt{m^2+8\lambda S})^2}{16\lambda^2}\frac{\partial C_{2-}}{\partial S}-\frac{N_c(m+\sqrt{m^2+8\lambda S})^2}{16\lambda^2}\frac{\partial C_{2+}}{\partial S}\,.
\end{aligned}
\tag{3.37}
$$

4. The field theory graphs

The entire lagrangian of the C-deformed field theory is not known. Hence it is not clear how to compute the correlation functions in presence of graviphoton background. In this section we make some assumptions about the lagrangian and demonstrate the difficulties arising from these assumptions.

According to the Dijkgraaf-Vafa conjecture the perturbative expansion of the graviphoton correction terms should be obtainable by computing the non-planar graphs of the theory. In [7, 8] a scheme was given for computing such graphs for matter in the adjoint representation of the gauge group. This scheme does not appear to be applicable in the case of matter in the fundamental representation since the scheme includes the selection rule that a non-vanishing graph with h holes should have gaugino insertions in $h-1$ of its holes. This selection rule originated from the requirement that the graphs be path-ordering independent. Basically, for a graph to be path-ordering independent one must have

$$\oint_{\gamma_i} p_\alpha = 0 \tag{4.1}$$

in that graph, where p_α is the world-sheet current of space-time supersymmetry and γ_i is the contour of the i-th hole. This was enforced by inserting $h-1$ gaugino insertions

$$\prod_{i=1}^{h-1}\left(\oint_{\gamma_i} W^\alpha p_\alpha\right)^2$$

and utilizing the fact that

$$\sum_{i=1}^{h}\oint_{\gamma_i} p_\alpha = 0\,. \tag{4.2}$$

However, (4.2) does not hold in the case of matter in the fundamental representation. This is most easily seen in the field theory limit, in which (4.2) takes the form [7]

$$\sum_{i=1}^{h}\sum_{I} s_I L_{Ii}\pi_\alpha^I = 0\,, \tag{4.3}$$

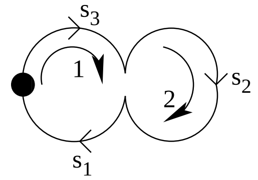
where the index I denotes the propagator and L_{Ii} is a matrix relating index-loop momenta to the propagator momenta. Unlike the double-line propagators of matter in the adjoint representation, propagators of matter in the fundamental representation have only a single color line. Therefore, whereas in the adjoint case each propagator is traversed twice in opposite directions and thus contributes two identical terms with opposite signs to the sum, so (4.3) is satisfied, in the fundamental case it is traversed once and obviously the sum can no longer vanish. Note that in the string theory description, similar subtleties may be encountered due to the way fundamental matter is engineered geometrically via singular Calabi-Yau compactification, or D-brane wrappings.

Thus we are forced to take a different approach. If one assumes that the lagrangian of the field theory is such that the graviphoton does not couple to the fields in a way that modifies the propagator of the chiral superfields, the techniques employed in [23] can still be used in the absence of gravity. Under such assumptions the graviphoton dependence will show up as a result of applying the chiral ring relations. The chiral superfield propagator is given by [23]

$$\frac{1}{p^2 + W^\alpha \pi_\alpha + m} = \int_0^\infty ds e^{-s(p^2 + W^\alpha \pi_\alpha + m)}\,, \tag{4.4}$$

where p and π_α are the bosonic and fermionic momenta, respectively, and using holomorphicity $\bar{m}$ has been taken to be 1. The vertices of the theory are read directly from the tree-level superpotential. The computation then proceeds by integrating over both the bosonic and fermionic loop-momenta. The bosonic integral is a simple gaussian integral yielding a determinant depending on the Schwinger parameters s_I while the fermionic one brings down insertions of W_α which combine using the chiral ring relations to form the graviphoton and glueball dependent terms multiplying a polynomial of the Schwinger parameters. According to the conjecture the fermionic and bosonic s-dependence should cancel leaving only a vector model computation. The W_α insertions must be path-ordered as these are no longer anti-commutative.

However, as we will soon demonstrate this approach fails. An example of this is one of the graphs for the correlation function $\langle M \rangle$



The bosonic integral is found to be

$$Z_B = \frac{1}{(4\pi)^4 (s_1 + s_3)^2 s_2^2}\,, \tag{4.5}$$

while the fermionic integration with the origin of path-ordering taken to be the M insertion yields by utilizing the chiral ring relations

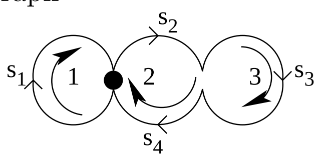
$$\begin{aligned} Z_F &= \frac{1}{16} s_2^2 \left[(s_3^2 + s_1^2)\, \mathrm{Tr}(W^2 W^2) + 2 s_1 s_3\, \mathrm{Tr}(W^\alpha W^2 W_\alpha) \right] = \\ &= -\frac{N_c}{32} s_2^2 (s_1 - s_3)^2 F_{\alpha\beta} F^{\alpha\beta}\,, \end{aligned} \tag{4.6}$$

whose s-dependence does not cancel with that of Z_B. But taking the path-ordering origin at the vertex one has

$$Z_F = -\frac{N_c}{32} s_2^2 (s_1 + s_3)^2 F_{\alpha\beta} F^{\alpha\beta} \tag{4.7}$$

leading to the exact cancellation of the s-dependence of this graph. One is drawn to conclude that this diagram should be taken to be zero by some selection rule in order to remove this ambiguity.

Other graphs also feature such problems. For example, the correlation function $\langle M^2 \rangle$ includes contribution from the graph



The bosonic integral in this case is

$$Z_B = \frac{1}{(4\pi)^6 s_1^2 (s_2+s_4)^2 s_3^2}\,, \tag{4.8}$$

while the fermionic integration with the origin of the path-ordering taken to be at the M^2 insertion is

$$Z_F = \frac{\pi^2}{4} s_1^2 s_3^2 (s_2-s_4)^2 F_{\alpha\beta}F^{\alpha\beta} S\,. \tag{4.9}$$

Taking the origin of the path-ordering at the other vertex yields the same result. It can be seen that again in this case the s-dependence does not cancel.

We conclude that either the field theory lagrangian must also be deformed in some way in addition to the C-deformation, or an appropriate scheme has to be developed within the framework of the above assumptions.

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